246 KSME International Journal, Vol. 13, No. 3, pp. 246~252, 1999

An Adjoint Variable Method for Design Sensitivity Analysis of Elastoplastic Structures

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(Received October 21, 1998)

Design sensitivity analysis of structural problems obeying an elastoplastic material behavior is developed using adjoint variable method. An elastoplastic constitutive equation with yield surface and kinematic hardening is considered to describe the material behavior. The traditional incremental procedure and its design variation need special treatments in order to predict the discontinuity of the structural response sensitivity because the contribution from the design sensitivity at the material transition point is lost during the calculation. In this study, discontinuities of the design variations at the material transition points are alleviated in the adjoint variable method. Analytical and numerical examples are used not only to demonstrate the developed sensitivity procedure but also to gain insights of numerical implementation for the design sensitivity analysis of the elastoplastic structure based on the adjoint variable method. The comparisons between adjoint variable and direct variation methods are also discussed.

Key Words: Design Sensitivity Analysis, Adjoint Variable Method, Elastoplasticity, Truss Structure, Finite Element Method, Structural Optimization.

1. Introduction

Design sensitivity coefficients are necessary in various fields such as the optimal design process, reliability analysis, probabilistic analysis, and the determination of relative importance of design variables(Kamart, 1993). Improved design can be achieved systematically by an engineer using the design sensitivity information. Because of the extension of the material usage from the elastic regime to the plastic regime, the discontinuous material behavior of the elastoplasticity becomes a significant consideration in design and analysis for practical applications.

In many practical applications, a piecewise linear continuous model approximates the elastoplastic constitutive law. For such a model, the design sensitivity coefficients are discontinuous at the material transition points that are located on the yield surface. Therefore the design sensitivity analysis (DSA) of elastoplastic structures is classified into discontinuous problem(Lee and Arora, 1995). The traditional incremental procedure and its design variation reveal difficulties in estimating the discontinuity of the structural response sensitivity at the material transition points because the contribution from the design sensitivity is lost during the calculation. DSA of elastoplastic structures has been carefully treated by using incremental form of direct variation method (DVM), where high computational accuracy of the displacement at the material transition points is necessary(Vidal and Haber, 1993; Ohsaki; and Arora, 1994; Ohsaki, 1997). Another DVM of elastoplastic structures overcoming the discontinuities at the material transition points was proposed by taking design variation of the response variables at a load level instead of their increments(Lee and Arora, 1995). DSA based on the so-called virtual distortion method with in-elastic and elastoplastic truss structure is presented recently(Kolakowski and Holnicki-Szulc, 1998).

When the dimensions of the design variables

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are larger than the numbers of response functions whose design sensitivity analysis is desired, the so-called adjoint variable method (AVM) is more efficient than DVM(Eaug et al., 1986). Therefore the purpose of this study is to develop AVM of DSA for an elastoplastic structure. The continuum formulations for the response and sensitivity analyses of a structural problem are used where only material nonlinearity is considered. The response analysis is performed using an incremental procedure that gives increments in the response variables. This is another way of overcoming the discontinuities in the design variations of the response variables at the material transition points of the elastoplastic structures. Since the adjoint equation derived here is nonlinear, an iteration process is necessary in the numerical calculation of adjoint variables or the tangent stiffness needs to be updated at each load step. An analytic example is illustrated to verify the developed theory and provide insights for implementation of the developed theory into computer programs. To show the numerical implementation of the developed method, an asymmetric three-bar truss structure is illustrated. Based on the experiences through this study, concluding remarks are followed.

2. Definition of Problem

2.1 Constitutive equation

Stress-strain relation of an elastoplastic model is often constructed on the basis of the yield criterion. In addition, many elastoplastic models use the fundamental assumption that the infinitesimal strain at load level t can be decomposed into elastic and plastic component as(Khan and Huang, 1995)

$$^te_{ij}=e_{ij}(^t\mathbf{u})=\frac{1}{2}(^tu_{i,j}+{}^tu_{j,i})={}^te^e_{ij}+{}^te^p_{ij} \quad (1)$$

where the right superscripts e and p indicate elastic and plastic parts, respectively, and $^t\mathbf{u}$ is the displacement field at load level t. The Cauchy stress can be determined using the generalized Hooke's law as

$$^t\sigma_{ij}=C_{ijkl}(^te_{kl}-{}^te^p_{kl});\ {}^te_{ij}={}^te_{ij}(^t\mathbf{u});$$

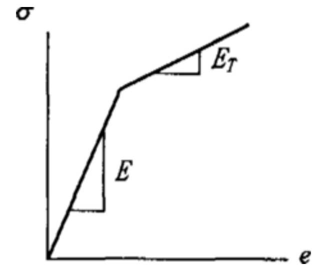

Fig. 1 Uniaxial stress-strain relationship of an elastoplastic material.

$$e^p_{ij}={}^te^p_{ij}(^t\mathbf{e}) \quad (2)$$

where C_{ijkl} denotes the Young's modulus of elasticity of a continuum. Note that a repeated index represents the summation convention over its range. A uniaxial constitutive equation of the rate-independent plasticity as shown in Fig. 1 is given as

$$^tF=|^t\sigma-c_K{}^te^p|-\sigma_Y;\ {}^te^p=\frac{^tP/A-\sigma_Y}{^cK}H(^tF)$$
$$=\frac{1}{E+c_K}(E^te-\sigma_Y)H(^tF) \quad (3)$$

where ^tF, $H(x)$, σ_Y and c_K denote the function of yielding point, the Heaviside unit step function, the size of the yield surface, and the kinematic hardening constant, respectively. It is noted that only the kinematic hardening is considered in this study.

2.2 Equilibrium equation

The virtual work equation governing the equilibrium state of a continuum body at load level t is given as

$$\int{}^t\sigma_{ij}e_{ij}(\delta^t\mathbf{u})\,dV=\int{}^tf_i\delta^tu_i dV$$
$$+\int{}^tT_i^0\delta^tu_i d\Gamma_T \ for\ \forall\delta^tu_i\in U(V),$$
$$^tu_i={}^tu_i^0\ on\ \Gamma_u\ and$$
$$^t\sigma_{ij}{}^tn_j={}^tT_i^0\ on\ \Gamma_T \quad (4)$$

where $e_{ij}(\delta^t\mathbf{u})=\delta^te_{ij}$ and $U(V)$ denotes a space of kinematically admissible displacements. Since Eq. (4) is nonlinear, incremental formulation of Eq. (4) is used to obtain a linearized equilibrium equation(Bathe, 1996) as

$$\int\sigma_{ij}e_{ij}(\delta\mathbf{u})\,dV=\int f_i\delta u_i dV$$

$$+\int T_i^0 \delta u_i d\Gamma_T \text{ for } \forall \delta^t u_i \in U(V),$$
$$u_i = u^0{}_i \ on \Gamma_u \ and$$
$$\sigma_{ij}{}^t n_j = T_i^0 \ on \Gamma_T. \quad (5)$$
$$\sigma_{ij} = C_{ijkl}(e_{kl} - e_{kl}^p);\ {}^t u_i = {}^{t-\Delta t}u_i + u_i;$$
$${}^t\sigma_{ij} = {}^{t-\Delta t}\sigma_{ij} + \sigma_{ij};\ {}^t f_i = {}^{t-\Delta t}f_i + f^i;$$
$${}^t T_i^0 = {}^{t-\Delta t}T_i^0 + T_i^0 \quad (6)$$

where u_i, σ_{ij}, f_i, $e_{ij}(\mathbf{u})$, *and* T_i^0 represent increments in the corresponding state fields. Now the linearized incremental equilibrium equation given in Eq. (5) can be solved for the increments of the state variables within the load increment. Note that since the left-hand side of the incremental equilibrium Eq. (5) depends on the solution variables of e_{ij}^p over the plastic regime, an iteration process with initial tangent stiffness is necessary or the tangent stiffness needs to be updated at each load increment.

3. Design Sensitivity Analysis

3.1 Design sensitivity analysis

Since DSA computes the rate of changes in a response due to the changes in a design variable b, DSA is defined as determination of the total variation of the response function, $\Psi({}^t u_i, b)$, with respect to the design variable as follows:

$$\text{Find } \tilde{\delta}\Psi = \frac{d\Psi}{db}\delta b \quad (7)$$

where it is assumed that the response function whose DSA is required can be expressed in terms of the design variables and the displacement field, which must satisfy the equilibrium Eq. (4).

3.2 Direct variation method

DSA of an elastoplastic structure has been presented using DVM(Vidal and Haber, 1993 ; Ohsaki and Arora, 1994; Ohsaki, 1997; Park and Choi, 1996; Tasy et al., 1993). Let ${}^r x_i$ be the coordinates in the fixed reference domain with the volume ${}^r V$ and its boundary ${}^r\Gamma$(Phelan and Haber, 1989; Arora et al., 1992). Applying the transformation $x_i \rightarrow {}^r x_i$ to Eq. (4), we have

$$\int {}^t\sigma_{ij} e_{ij}(\delta^t \mathbf{u})\, {}^r J dV = \int {}^t f_i \delta^t u_i J^r dV$$
$$+ \int {}^t T_i^0 \delta^t u_i J_\Gamma^r d\Gamma_T \ for\ \forall \delta^t u_i \in U({}^r V),$$
$${}^t u_i = {}^t u_i^0 \Gamma_u \ and\ {}^t\sigma_{ij}{}^t n_j = {}^t T_i^0 \ on\ {}^r\Gamma_T;$$
$${}^t e_{ij} = \frac{1}{2}({}^t u_{i;m} x_{m,j} + {}^t u_{j;m} x_{m,i}). \quad (8)$$

The total design variations of strain and stress are given as

$$\tilde{\delta}^t e_{ij} = \bar{\delta}^t e_{ij} + \bar{\bar{\delta}}^t e_{ij};\ \bar{\delta}^t e_{ij} = e_{ij}(\bar{\delta}^t \mathbf{u});$$
$$\tilde{\delta}(\delta^t e_{ij}) = e_{ij}(\bar{\delta}(\delta^t \mathbf{u})) + \bar{\bar{\delta}} e_{ij}(\delta^t \mathbf{u}), \quad (9)$$
$$\tilde{\delta}^t \sigma_{ij} = \bar{\delta}^t \sigma_{ij} + \bar{\bar{\delta}}^t \sigma_{ij};$$
$$\bar{\delta}^t \sigma_{ij} = C_{ijkl}\{e_{kl}(\bar{\delta}^t \mathbf{u}) - \bar{\delta}^t e_{kl}^p\};\ \bar{\bar{\delta}}^t \sigma_{ij}$$
$$= \bar{\bar{\delta}} C_{ijkl}({}^t e_{kl} - {}^t e_{kl}^p) + C_{ijkl}(\bar{\bar{\delta}}^t e_{kl} - \bar{\bar{\delta}}^t e_{kl}^p) \quad (10)$$

where $\bar{\delta}$ and $\bar{\bar{\delta}}$ denote implicit and explicit design variations, respectively(Lee and Arora, 1995). The design variation of the plastic strain given in Eq. (2) becomes

$$\tilde{\delta}^t e_{ij}^p = \bar{\delta}^t e_{ij}^p + \bar{\bar{\delta}}^t e_{ij}^p;\ \bar{\delta}^t e_{ij}^p = {}^t e^p_{ij, {}^t e_{kl}} \bar{\delta}^t e_{kl}. \quad (11)$$

Now in order to calculate $\bar{\delta}^t \mathbf{u}$ using DVM, total design variation of the virtual work Eq. (4) is taken as

$$\int \bar{\delta}^t \sigma_{ij} e_{ij}(\delta^t \mathbf{u}) J^r dV = \int \bar{\bar{\delta}}({}^t f_i J)\, \delta^t u_i dV$$
$$+ \int \bar{\bar{\delta}}({}^t T_i^0 J_\Gamma)\, \delta^t u_i d\Gamma_T - \int \bar{\bar{\delta}}\{{}^t \sigma_{ij} e_{ij}(\delta^t \mathbf{u}) J\}^r dV$$
$$\text{for } \forall \delta^t u_i \in U({}^r V) \quad (12)$$

based an the fact that $\bar{\delta}(\delta^t \mathbf{u})$ is kinematically admissible virtual displacement. Note that the design sensitivity equation derived by DVM of Eq. (12) has same operators as those in the linearized incremental equilibrium Eq. (5). However, it is important to note that the left-hand side of the design sensitivity equation depends on the solution variables of $\bar{\delta}^t e_{ij}^p$.

3.3 Adjoint variable method

Consider a response function defined in the unit volume and transformed into the reference domain as

$$\Psi = \int \frac{1}{J} G({}^t u_i, b) J^r dV. \quad (13)$$

To develop AVM, we define an augmented functional as

$$L = \int \frac{1}{J} G({}^t u_i, b) J^r dV - \int {}^t \sigma_{ij} e_{ij}({}^t_a \mathbf{u}) J^r dV$$
$$+ \int {}^t f_i {}^t_a u_i J^r dV + \int {}^t T_{ia}^{0t} u_i J_\Gamma^r d\Gamma_T \quad (14)$$

where kinematically admissible field ${}^t_a u_i$, the so-called adjoint variable, is introduced and will be determined later. Using the variational principle of DSA(Arora and Cardoso, 1992), we have

$$\begin{aligned}\bar{\delta}_\Psi &\equiv \bar{\bar{\delta}} L \\ &= \int \bar{\delta} G({}^t u, b)\, {}^r dV - \int \bar{\bar{\delta}}\, {}^t\sigma_{ij} e_{ij}({}^t_a \mathbf{u}) J^r dV \\ &- \int {}^t\sigma_{ij} \bar{\bar{\delta}} e_{ij}({}^t_a \mathbf{u}) J^r dV \\ &- \int {}^t\sigma_{ij} e_{ij}({}^t_a \mathbf{u})\, \bar{\bar{\delta}} J^r dV + \int \bar{\bar{\delta}}({}^t f_i J)\, {}^t_a u_i^r dV \\ &+ \int \bar{\bar{\delta}}({}^t T_i^0 J_\Gamma)\, {}^t_a u_i^r d\Gamma_T. \end{aligned} \tag{15}$$

Since $\bar{\delta} L = 0$, the adjoint equation is given as follows:

$$\begin{aligned}&\int {}^t_a\sigma_{ij}\, {}^t e_{ij}(\bar{\delta}\, {}^t\mathbf{u}) J^r dV = \int G_{,{}^t u_i} \bar{\delta}\, {}^t u_i{}^r dV \\ &+ \int {}^t_a\sigma_{ij}\, {}^t e^p_{ij,{}^t e_{kl}}\, {}^t e_{kl}(\bar{\delta}\, {}^t\mathbf{u}) J^r dV \\ &\text{for } \forall\, \bar{\delta}\, {}^t u_i \in U(V) \end{aligned} \tag{16}$$

where ${}^t_a\sigma_{ij} = C_{ijkl}\, {}^t_a e_{kl}$ and ${}^t_a u_i \in U(V)$. Solving Eq. (16) to adjoint displacement ${}^t_a u_i$, we can compute the design sensitivity coefficient of the response function of Eq. (15) by substituting the adjoint displacement into Eq. (15). Note that the adjoint variable eq. (16) has the same operators as those in the linearized incremental equilibrium Eq. (5). Since the right-hand side of Eq. (16) has unknown variable ${}^t_a u_i$, an iteration process is necessary or the tangent stiffness must to be updated at each load increment to solve it.

4. Examples

4.1 Analytical example : linear elastoplastic rod

Consider a linear elastoplastic rod subjected to an axial load as shown in Fig. 2(Lee and Arora, 1995). DSA of the tip displacement is illustrated analytically, where the design variable is $\mathbf{b} = [A]$, i.e., the cross-sectional area. Using Dirac delta function, $\check{\delta}(\xi)$, the response functional can be given as

$$\Psi = \int_0^1 {}^t u(\xi)\, \check{\delta}(\xi - 1)\, d\xi. \tag{17}$$

(i) When the load ${}^t P$ is in the elastic range (i.e., ${}^t F < 0$), strain, equilibrium equation and tip displacement are given as

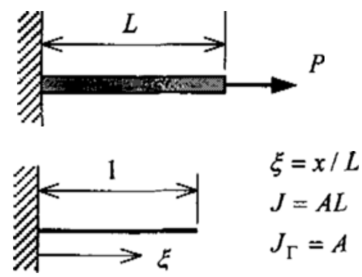

Fig. 2 Linear elastoplastic rod and its transformation into the reference domain.

$$\begin{aligned}{}^t e &= {}^t u_{,x} = \frac{1}{L}\, {}^t u_{,\xi};\ \int_0^1 \left\{ \frac{E}{L^2} u_{,\xi} (\delta^t u)_{,\xi} AL \right\} d\xi \\ &= {}^t P \delta^t u(1);\ {}^t u(L) = \frac{{}^t PL}{EA}. \end{aligned} \tag{18}$$

DVM of sensitivity equation (12) and its solution are given as

$$\begin{aligned}&\bar{\delta}\, {}^t e = \frac{1}{L} \bar{\delta}\, {}^t u_{,\xi};\ \int_0^1 \left\{ \frac{E}{L^2} \bar{\delta} u_{,\xi} (\delta^t u)_{,\xi} AL \right\} d\xi = \\ &-\int_0^1 \left\{ \frac{E}{L^2}\, {}^t u_{,\xi} (\delta^t u)_{,\xi} L \bar{\bar{\delta}} A \right\} d\xi; \\ &\bar{\delta}\, {}^t u(L) = -\frac{{}^t PL \bar{\delta} A}{EA^2}. \end{aligned} \tag{19}$$

Adjoint equation and adjoint variable of AVM can be evaluated from Eqs. (16) and (17) as follows:

$$\begin{aligned}&\int_0^1 \left\{ \frac{E}{L^2}\, {}^t_a u_{,\xi} \bar{\delta}\, {}^t u_{,\xi} AL \right\} d\xi = \int_0^1 \left\{ \frac{1}{AL^2} \check{\delta}(\xi - 1) \right. \\ &\left. \bar{\delta}\, {}^t u AL \right\} d\xi;\ {}^t_a u = \frac{L\xi}{EA} \end{aligned} \tag{20}$$

where it is observed that the pseudo-load of the adjoint structure is unit. Now the design variation of the response functional Eq. (17), is given from Eq. (15) as follows:

$$\bar{\delta}\Psi = -\int_0^1 \frac{E}{L}\, {}^t u_{,\xi}\, {}^t_a u_{,\xi} d\xi \bar{\delta} A = -\frac{{}^t PL}{EA^2} \bar{\delta} A. \tag{21}$$

Note that the design variation of the tip displacement given by DVM and AVM is the same as the direct design variation of the analytical solution of Eq. (18).

(ii) When the load ${}^tP$ is in the plastic range (i. e., ${}^tF \geq 0$), the equilibrium equation and its solution by integration by parts are given as

$$\int_0^1 \left\{ \frac{E}{L^2} {}^tu_{,\xi} \delta^t u_{,\xi} AL \right\} d\xi = {}^tP \delta^t u(1)$$

$$+ \int_0^1 \left\{ \frac{E}{L} \frac{1}{E + c_K} \left(\frac{E}{L} {}^tu_{,\xi} - \sigma_Y \right) \delta^t u_{,\xi} AL \right\} d\xi, \tag{22}$$

$${}^tu(L) = \frac{{}^tPL}{A} \left(\frac{1}{E} + \frac{1}{c_K} \right) - \frac{L}{c_K} \sigma_Y. \tag{23}$$

DVM of sensitivity Eq. (12) and its solution are given as

$$\int_0^1 \left\{ \frac{E}{L^2} \bar{\delta}^t u_{,\xi} (\delta^t u)_{,\xi} AL \right\} d\xi$$

$$= -\int_0^1 \left\{ \frac{E}{L^2} {}^tu_{,\xi} (\delta^t u)_{,\xi} L \bar{\delta} A \right\} d\xi$$

$$+ \bar{\delta} \int_0^1 \left\{ \frac{E}{L^2} {}^te^p (\delta^t u)_{,\xi} AL \right\} d\xi, \tag{24}$$

$$\bar{\delta}^t u(L) = -\left(\frac{1}{E} + \frac{1}{c_K} \right) \frac{{}^tPL\bar{\delta}A}{A^2}. \tag{25}$$

Since plastic strain is implicit in design, design sensitivity Eq. (16) of AVM and adjoint variable are given as follows:

$$\int_0^1 \left(\frac{E}{L^2} {}^t_a u_{,\xi} \bar{\delta}^t u_{,\xi} AL \right) d\xi$$

$$= \int_0^1 \frac{1}{AL} \delta(\xi - 1) \bar{\delta}^t AL d\xi$$

$$+ \int_0^1 \left(\frac{E}{L^2} {}^t_a u_{,\xi} \frac{E}{E + c_K} \bar{\delta}^t u_{,\xi} AL \right) d\xi, \tag{26}$$

$${}^t_a u = \left(\frac{1}{E} + \frac{1}{c_K} \right) \frac{L}{A} \xi. \tag{27}$$

Thus the design variation of the response functional is given as

$$\bar{\delta}\Psi = -\int_0^1 \frac{E}{L} {}^tu_{,\xi} {}^t_a u_{,\xi} d\xi \bar{\delta} A$$

$$= -\left(\frac{1}{E} + \frac{1}{c_K} \right) \frac{{}^tPL\bar{\delta}A}{A^2} \tag{28}$$

which is identical to the design variation of the analytical displacement of Eq. (23). Note that the design variation of the tip displacement at load level t is evaluated without the aid of DSA of the response corresponding to the yield load (discontinuous point) in both DVM and AVM. This is an important observation because it shows that the sensitivity analysis at the yield point is not needed to calculate sensitivities in the plastic regime.

4.2 Numerical example : three-bar truss structure

Numerical example is provided for an asymmetric three-bar truss structure subjected to a load $P > P_Y$ as shown in Fig. 3, where P_Y denotes the critical load corresponding to the yield point for member 2. The problem is to find the design sensitivity of the x-displacement at node 4 due to the cross-sectional area variation of the member 2, i.e., $b = A_2$. The dimension and material properties of the problem are

$E = 100\ GPa$; $E_t = 10\ GPa$; $\sigma_Y = 200\ MPa$;
$P = 90\ kN$; $\Delta P = 10\ N$; $\alpha = 60°$; $\beta = 30°$
$A_1 = A_2 = A_3 = 1 \times 10^{-4} \mathrm{m}^2$; $L = 1\mathrm{m}$

A_1, A_2, and A_3 represent cross-sectional areas of members 1, 2, and 3, respectively.

Figure 4 shows the load-displacement relationship and Fig. 5 shows the design sensitivity coefficients of the x-displacement due to the design variation of A_2. Member 2 yields first at load level around 34.6 kN and then members 3 and 1 at loads around 44.2 kN and 76.6 kN, respective-

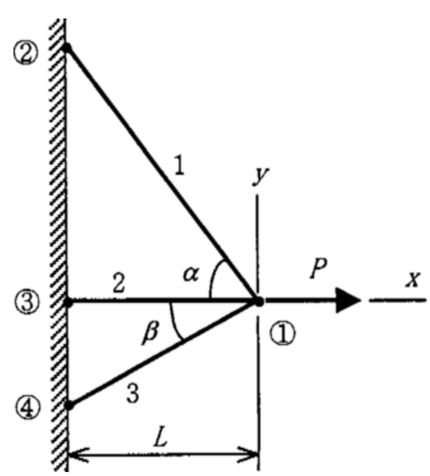

Fig. 3 Asymmetric three-bar truss structure.

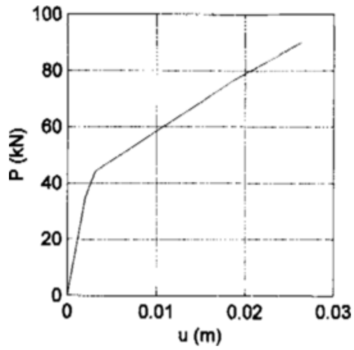

Fig. 4 Load-displacement plot.

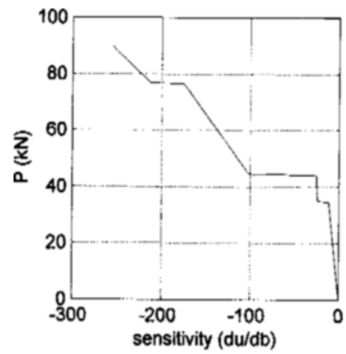

Fig. 5 Design sensitivity of the x-displacement of node 1.

ly. The accuracy of the design sensitivity coefficients evaluated by the developed method is compared with those by the central finite difference method with 1% perturbation of b as given in Table 1. In the presentation of the results, R indicates the ratio of the design sensitivity coefficients predicted by the central finite difference method ($\Delta u/\Delta b$) to those by AVM (du/db) as follows:

$$R=\frac{\Delta u/\Delta b}{du/db}\times 100(\%). \tag{29}$$

$R=100\%$ means that the design sensitivity coefficient predicted by AVM matches exactly with it evaluated by the central finite difference method. It can be observed that the design sensitivity coefficients are in good agreement with those obtained using the overall central finite difference method. However, the central finite difference gives inaccurate design sensitivity coefficients around the material transition points as shown in Table 1. This is attributed to the errors in the

Table 1 Design sensitivity coefficients of x-displacement of member 1 and their comparison with central finite difference method.

$P(kN)$	$u(m)$	du/db*	$\Delta u/\Delta b$	$R(\%)$
30.0	0.0017	−10.0000	−10.0000	100.00
34.0	0.0020	−11.3333	−11.3334	100.00
34.5	0.0020	−11.5000	−14.5050	126.13
35.0	0.0020	−24.0749	−23.7040	98.46
40.0	0.0026	−24.7994	−25.8533	104.25
43.5	0.0031	−25.2748	−24.3588	96.38
44.0	0.0031	−25.3578	−30.4873	120.23
44.5	0.0033	−101.7359	−102.1525	100.41
50.0	0.0059	−114.2084	−118.4113	103.68
55.0	0.0083	−125.6501	−126.8387	100.95
60.0	0.0107	−137.0816	−141.8282	103.46
65.0	0.0131	−148.5233	−148.0698	99.69
70.0	0.0155	−160.0316	−158.7037	99.17
75.0	0.0179	−171.3581	−173.5995	101.31
76.0	0.0183	−173.6680	−172.7173	99.45
76.5	0.0186	−174.7934	−183.3698	104.91
77.0	0.0189	−212.7827	−212.6301	99.93
80.0	0.0206	−222.7827	−226.6304	99.93
85.5	0.0238	−241.1161	−233.1849	96.71
90.0	0.0264	−256.1161	−248.1854	96.90

* Sensitivity coefficients computed by AVM

finite difference calculations due to the discontinuities at the transition points. The developed AVM provides however quite reasonable design sensitivity coefficient for all load ranges as shown in Fig. 5.

5. Discussion and Concluding Remarks

Adjoint variable method for DSA of elastoplastic structural problem is developed and demonstrated. The adjoint equation is obtained by taking implicit design variation of the augmented functional defined. In this way, discontinuities of the design variations at the material transition points do not affect DSA at other points, i. e., no special treatment is needed to overcome discontinuities. Analytical and numerical examples are used to demonstrate the developed AVM procedure and gain insights for their numerical implementations. It is observed that an iteration procedure in AVM is necessary to compute sensitivity expressions or the tangential stiffness matrix needs to be updated at each load increment. Moreover, it is important to note that AVM can be implemented for elastoplastic structural problems as easily as DVM. Thus when the dimensions of design variables are larger than the number of response functions whose design sensitivity analysis is desired, the developed AVM is more efficient than DVM.

Acknowledgement

The author wishes to acknowledge the financial support of Hanyang University, Korea, made in the program year of 1997.

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